Digital Resonant Current Controllers for Voltage Source Converters

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“Doctor Europeus” mention
Outline

1. Introduction
2. Effects of Discretization Methods on the Performance of Resonant Controllers
3. High Performance Digital Resonant Current Controllers Implemented with Two Integrators
5. Conclusions
Outline

1. Introduction
   - Plant Model for Current-Controlled VSCs
   - Review of Current Controllers for VSCs
   - Objectives

2. Effects of Discretization Methods on the Performance of Resonant Controllers

3. High Performance Digital Resonant Current Controllers Implemented with Two Integrators


5. Conclusions
Plant Model for Current-Controlled VSCs

Model suitable for active filters, active rectifiers, adjustable speed drives (decoupled back EMF), etc.

\( v_{ac} \): grid voltage, back EMF of electric machine, etc.

**L filter**

\[
G_L(s) = \frac{I(s)}{V_C(s)} = \frac{1}{sL_F + R_F}
\]

**LCL filter**

\[
G_{LCL}^C(s) = \frac{I_C(s)}{V_C(s)}; \quad G_{LCL}^G(s) = \frac{I_G(s)}{V_C(s)}
\]
**Plant Model for Current-Controlled VSCs II**

LCL filters behave as $L$ at frequencies lower than approx. $f_{\text{res}}$:

\[
G_{LCL}^C(s) \approx G_{LCL}^G(s) \approx G_L(s) = \frac{1}{sL_F + R_F}
\]
Digital Resonant Current Controllers for Voltage Source Converters

Introduction

Plant Model for Current-Controlled VSCs

Plant Model for Current-Controlled VSCs III

Complete block diagram

Simplified block diagram

- $G_C$: current controller
- $G_L$: L filter
- $G_{PL}$: plant model

$$G_{PL}(s) = e^{-sT_s} \left( \frac{1 - e^{-sT_s}}{s} \right) \frac{1}{sL_F + R_F} \frac{G_L(s)}{G_{PL}(s)}$$

$$G_{PL}(z) = \mathcal{Z}\left\{ \mathcal{L}^{-1}[G_{PL}(s)] \right\} = \frac{z^{-2}}{R_F} \frac{1 - \rho^{-1}}{1 - z^{-1} \rho^{-1}} \text{ where } \rho = e^{\frac{RF T_s}{L_F}}$$
Plant Model for Current-Controlled VSCs

**Complete block diagram**

- $G_C(s)$: current controller
- $G_L(s)$: L filter
- $G_{PL}(s)$: plant model

**Simplified block diagram**

\[
G_{PL}(s) = \frac{e^{-sT_s} \cdot \text{Comp. delay}}{1 - e^{-sT_s}} \cdot \frac{\text{ZOH (PWM)}}{s} \cdot \frac{G_L(s)}{sL_F + R_F}
\]

\[
G_{PL}(z) = \mathcal{Z}\left\{\mathcal{L}^{-1}\left[G_{PL}(s)\right]\right\} = \frac{z^{-2}}{R_F} \cdot \frac{1 - \rho^{-1}}{1 - z^{-1} \rho^{-1}} \quad \text{where} \quad \rho = e^{\frac{R_F T_s}{L_F}}
\]


**Hysteresis Control**

- **Simplicity**
- **Unconditioned stability**
- **Very fast response**
- **Good accuracy**

- **Variable switching frequency** (resonances, filters, power losses, ripple...)
- **Interference** among phases
- **Inability to perform selective control**
- **Dependence on converter topology**
- **Analog comparators**
Hysteresis Control

- **Simplicity**
- **Unconditioned stability**
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Digital Resonant Current Controllers for Voltage Source Converters

Introduction

Review of Current Controllers for VSCs

## Deadbeat Control

$$\frac{G_{DB}(z) \cdot G_{PL}(z)}{1 + G_{DB}(z) \cdot G_{PL}(z)} = z^{-2} \Rightarrow G_{DB}(z) = \frac{R_F}{1 - \rho^{-1}} \cdot \frac{1 - z^{-1} \rho^{-1}}{1 - z^{-2}}$$

- **Simplicity**
- Theoretically, **fastest** transient among digital controllers
- Sensitiveness to deviations in **plant parameters**
- Need for $\nu_{ac}$ **feedforward**
- Sensitiveness to measurement **noise**
- Need for **dead-times** compensation
- Closed-loop steady-state error
**PI Control in SRF**

**Conventional PI controller**

\[ i_{dq}^* \rightarrow k_{Ph} \rightarrow k_{1h} \rightarrow \frac{1}{s} \rightarrow \frac{1}{L_Fs + R_F} \rightarrow jh\omega_1 L_F \rightarrow i_{dq} \]

**PI Controller with cross-coupling decoupling (PICCD)**

\[ i_{dq}^* \rightarrow k_{Ph} \rightarrow k_{1h} \rightarrow \frac{1}{s} \rightarrow \frac{1}{L_Fs + R_F} \rightarrow jh\omega_1 L_F \rightarrow i_{dq} \]

**Complex-vector PI controller**

\[ i_{dq}^* \rightarrow L_F \rightarrow jh\omega_1 L_F \rightarrow \frac{1}{s} \rightarrow \frac{1}{L_Fs + R_F} \rightarrow jh\omega_1 L_F \rightarrow i_{dq} \]
**Repetitive Controllers**

**Common characteristics**
- Tracking of *multiple* harmonics by a simple scheme
- Difficult *frequency adaptation*
- Same *parameters* for all peaks

**Recursive form**
- No *selectiveness*
- Steady-state error
- Difficult *tuning*

**DFT-based**
- Selectiveness
- No steady-state error
Introduction

Review of Current Controllers for VSCs

Proportional+Resonant (PR) Controllers

Equivalent to a **conventional PI** in **positive-sequence SRF** + another one in **negative-sequence SRF**

Conventional PI:

\[
G_{PIh}(s) = k_{P_h} + \frac{k_{I_h}}{s}
\]

Positive-sequence SRF:

\[
G_{PIh}^+(s) = G_{PIh}(s - j\omega_1) = k_{P_h} + \frac{k_{I_h}}{s - j\omega_1}
\]

Negative-sequence SRF:

\[
G_{PIh}^-(s) = G_{PIh}(s + j\omega_1) = k_{P_h} + \frac{k_{I_h}}{s + j\omega_1}
\]

\[
G_{PRh}(s) = G_{PIh}^+(s) + G_{PIh}^-(s) = 2k_{P_h} + 2k_{I_h} \frac{s}{s^2 + h^2\omega_1^2} = K_{P_h} + K_{I_h} \frac{s}{s^2 + h^2\omega_1^2}
\]

Delay compensation:

\[
G_{PRh}^d(s) = K_{P_h} + K_{I_h} \frac{s \cos(\phi'_h) - h\omega_1 \sin(\phi'_h)}{s^2 + h^2\omega_1^2}
\]

Total controller:

\[
G_C(s) = \sum_{h} G_{PRh}^d(s) = \sum_{h} K_{P_h} + \sum_{h} K_{I_h} R_{1h}^d(s)
\]
### Proportional+Resonant (PR) Controllers

Equivalent to a **conventional PI** in **positive-sequence SRF** + another one in **negative-sequence SRF**

Conventional PI:

\[
G_{\text{PI}_h}(s) = k_{\text{P}_h} + \frac{k_{\text{I}_h}}{s}
\]

\[
\begin{align*}
G_{\text{PI}_h}^+(s) &= G_{\text{PI}_h}(s - jh\omega_1) = k_{\text{P}_h} + \frac{k_{\text{I}_h}}{s-jh\omega_1} \\
G_{\text{PI}_h}^-(s) &= G_{\text{PI}_h}(s + jh\omega_1) = k_{\text{P}_h} + \frac{k_{\text{I}_h}}{s+jh\omega_1}
\end{align*}
\]

\[
G_{\text{PR}_h}(s) = G_{\text{PI}_h}^+(s) + G_{\text{PI}_h}^-(s) = 2k_{\text{P}_h} + 2k_{\text{I}_h} \frac{s}{s^2 + h^2\omega_1^2} = K_{\text{P}_h} + K_{\text{I}_h} \frac{s}{s^2 + h^2\omega_1^2}
\]

Delay compensation:

\[
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\]

Total controller:

\[
G_{\text{C}}(s) = \sum_{h} G_{\text{PR}_h}^d(s) = \sum_{h} K_{\text{P}_h} + \sum_{h} K_{\text{I}_h} R_{\text{1}_h}^d(s)
\]
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Proportional+Resonant (PR) Controllers

Equivalent to a conventional PI in positive-sequence SRF + another one in negative-sequence SRF

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$$G_{PIh}(s) = k_{P_h} + \frac{k_{Ih}}{s}$$

$$G_{PRh}(s) = G_{PIh}^+(s) + G_{PIh}^-(s) = 2k_{P_h} + 2k_{Ih} \frac{s}{s^2 + h^2\omega_1^2} = K_{P_h} + K_{Ih} \frac{s}{s^2 + h^2\omega_1^2}$$

Delay compensation:

$$G_{PRh}^d(s) = K_{P_h} + K_{Ih} \frac{s \cos(\phi'_h) - h\omega_1 \sin(\phi'_h)}{s^2 + h^2\omega_1^2}$$

Total controller:

$$G_C(s) = \sum_{h} G_{PRh}^d(s) = \sum_{h} K_{P_h} + \sum_{h} K_{Ih} R_{1h}^d(s)$$
Analysis and design of PR Controllers

**Open-loop**

- \( K_{PT} \rightarrow f_c, \ PM_P \)
- \( K_{I_h} \rightarrow \) bandwidth around \( hf_1 \)
- \( hf_1 < f_c \ \forall h \) such that \( \phi'_h = 0 \)
- \( \phi'_h \rightarrow \) anomalous peaks & stability

\[
G_C(s) = K_{PT} + \sum_{h=1}^{n_h} K_{I_h} R^{d}_{1h}(s)
\]

**Closed-loop**

- \( \phi'_h = 0 \)
- 2 samples \( \phi'_h \)
Vector Proportional+Integral (VPI) Controllers

Equivalent to a complex-vector PI in positive-sequence SRF + another one in negative-sequence SRF

\[
G_{VPI_h}(s) = G_{cPI_h}(s - jh\omega_1) + G_{cPI_h}(s + jh\omega_1) = 
\]

\[
= K_{P_h} \frac{s^2}{s^2 + h^2\omega_1^2} + K_{I_h} \frac{s}{s^2 + h^2\omega_1^2} = K_h \frac{s (s L_F + R_F)}{s^2 + h^2\omega_1^2}
\]

Delay compensation: \[
G_{VPI_h}^d(s) = K_h \frac{(s L_F + R_F) [s \cos(\phi'_h) - h\omega_1 \sin(\phi'_h)]}{s^2 + h^2\omega_1^2} = 
\]

\[
= K_{P_h} \frac{s^2 \cos(\phi'_h) - s h\omega_1 \sin(\phi'_h)}{s^2 + h^2\omega_1^2} + K_{I_h} \frac{s \cos(\phi'_h) - h\omega_1 \sin(\phi'_h)}{s^2 + h^2\omega_1^2}
\]
Vector Proportional+Integral (VPI) Controllers

Equivalent to a complex-vector PI in positive-sequence SRF + another one in negative-sequence SRF

\[
G_{VPI_h}(s) = G_{cPI_h}(s - j\omega_1) + G_{cPI_h}(s + j\omega_1) = \\
K_p \frac{s^2}{s^2 + h^2\omega_1^2} + K_i \frac{s}{s^2 + h^2\omega_1^2} = K_h \frac{s (s L_F + R_F)}{s^2 + h^2\omega_1^2}
\]

Delay compensation:

\[
G_{VPI_h}^d(s) = K_h \frac{(s L_F + R_F) [s \cos(\phi_h') - h\omega_1 \sin(\phi_h')]}{s^2 + h^2\omega_1^2} = \\
K_p \frac{s^2 \cos(\phi_h') - s h\omega_1 \sin(\phi_h')}{s^2 + h^2\omega_1^2} + K_i \frac{s \cos(\phi_h') - h\omega_1 \sin(\phi_h')}{s^2 + h^2\omega_1^2}
\]
Analysis and design of VPI Controllers

- \( K_h \rightarrow \text{bandwidth around } hf_1 \)
- \( \phi_h' \rightarrow \text{anomalous peaks & stability (only required at } \uparrow hf_1/f_s) \)
Main Objectives of this PhD Thesis

- To provide an in-depth study and comparison of the effects of discretization strategies.

- To develop optimized discrete-time implementations with a good tradeoff between
  - accuracy
  - resource-consumption & simplicity

- To propose an analysis and design methodology for resonant controllers by means of Nyquist diagrams, suitable for cases with more than one cross-over frequency.
### Digital Implementations

1. Discretization of continuous **transfer function** (by Tustin, zero-pole matching, etc.)

2. Two discrete **integrators**
   - Direct int.: Forward Euler   Feedback int.: Backward Euler (f&b)
   - Direct int.: Backward Euler Feedback int.: Backward Euler + $z^{-1}$ (b&b)
   - Direct int.: Tustin Feedback int.: Tustin (t&t)

---

#### PR

- Input: $K_p h$
- Output: $\frac{1}{s}$
- $h^2 \omega^2$
- $h \omega$

#### VPI

- Input: $K_i h$
- Output: $\frac{1}{s}$
- $h^2 \omega^2$
- $h \omega$
Resonant Poles Displacement ($f_s = 10$ kHz)
Resonant Poles Displacement ($f_s = 10$ kHz)

- A (f)
- B (b)
- C ($t$, $t$&t)
- D (f&b, b&b)
- E (zoh, foh, tp, zpm, imp)

![Graph showing resonant poles displacement with frequency (Hz) and magnitude (dB) on the axes.](image-url)
Resonant Poles Displacement (variable $f_s$)

Group C
(t, t&t)

Group D
(f&b, b&b)

Group E
(zoh, foh, tp, zpm, imp)
Effects on Zeros of $R_{1h}(s)$ (E methods)
Effects on Zeros of $R_{1h}(s)$ (E methods)

Better stability at high frequencies
Effects on Zeros of $R_{1h}(s)$ (E methods)

- $R_{1h}(s)$
- $R_{1h}^{\text{imp}}(z)$
- $R_{1h}^{\text{zoh}}(z) = R_{1h}^{\text{zpm}}(z)$

Better stability at high frequencies
Worse stability at high frequencies

Graph showing magnitude and phase vs. frequency.
Effects on Zeros of $R_{1h}(s)$ (E methods)

$R_{1h}(s)$

$R_{1h}^{imp}(z)$

$R_{1h}^{zoh}(z)=R_{1h}^{zpm}(z)$

$R_{1h}^{tp}(z) \approx R_{1h}^{foh}(z)$

Worse stability at high frequencies

Better stability at high frequencies

Same phase response as $R_{1h}(s)$

6.3°
Effects on Zeros of $R_{1h}(s)$ (D methods)
Effects on Zeros of $R_{1h}(s)$ (D methods)
Effects on Zeros of $R_{1h}(s)$ (D methods)
Effects on Zeros of $R_{2h}(s)$ (E methods)
Effects on Zeros of $R_{2h}(s)$ (E methods)

- Low gain at high frequencies
Effects on Zeros of $R_{2h}(s)$ (E methods)

- $R_{2h}(s)$
- $R_{2h}^{\text{imp}}(z)$
- $R_{2h}^{\text{zoh}}(z)$

Low gain at high frequencies

Zoom: 6.2°
Effects on Zeros of $R_{2h}(s)$ (E methods)

- $R_{2h}(s)$
- $R_{2h}^{imp}(z)$
- $R_{2h}^{zoh}(z)$

$R_{2h}(z) \approx R_{2h}^{zpm}(z) \approx R_{2h}^{foh}(z)$

Low gain at high frequencies
Effects on Zeros of $G_{VPI_h}(s)$ (D methods)
Effects on Zeros of $G_{VPIh}(s)$ (D methods)
Effects on Zeros of $G_{VPIh}(s)$ (D methods)
Effects on Delay Compensation (E methods)
Effects on Delay Compensation (D methods)
## Optimum Discrete-Time Implementations

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<th>No frequency adaptation</th>
<th>Frequency adaptation</th>
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<td><strong>No delay</strong></td>
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<td>$G_{VPIh}^d(z) = K_{P_h} R_{2h}^{foh,tp,zpm}(z) + K_{I_h} R_{1h}^{imp,foh,tp}(z)$</td>
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<tr>
<td><strong>Delay</strong></td>
<td>$G_{PRh}^d(z)$</td>
<td>$G_{VPIh}^d(z) = K_{P_h} R_{2h}^{foh,tp}(z) + K_{I_h} R_{1h}^{imp,foh,tp}(z)$</td>
</tr>
<tr>
<td>comp.</td>
<td></td>
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</table>

* Requires the on-line computation of $\cos(h\omega_1 T_s)$ terms as $h\omega_1$ varies.
Experimental Setup (APF)
Experimental Setup (APF)

- **$i_L$: uniform spectrum**

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<th>CH2</th>
<th>CH3</th>
<th>CH4</th>
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<td>THD-R</td>
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</table>

**VSC Diagram**

- $v_S$: Source Voltage
- $L_s$: Source Inductance
- $i_s$: Source Current
- PCC: Power Connection Center
- $i_L$: Load Current
- $v_{dc}$: DC Link Voltage
- $C$: DC Link Capacitor
- $L_F$: Filter Inductance
- $R_F$: Filter Resistance
- $v_{ac}$: AC Source Voltage
- $i_F$: Filter Current
- $i_{F1}$: Phase 1 Filter Current
- $i_{Lh}$: Load Harmonic Current
- DSP: Digital Signal Processor
- PLL: Phase Lock Loop
- HARMONIC DETECTION
- CURRENT CONTROLLER

**dSPACE DS1104**

**Programmable AC Load**
Resonant Poles Displacement I
Resonant Poles Displacement II

$K_{PT}$

$G_{PR_h}(z)$

$G_{f&b}(z)$
Discrete-Time Delay Compensation

\[ G_{PR_h}^{imp}(z) \]

\[ G_{VPI_h}^{imp-tp}(z) \]
Conclusions of Chapter II

- An exhaustive study and comparison of discretization techniques applied to resonant controllers has been presented, in terms of their effects on
  - steady-state error
  - stability

- It has been shown that the choice of discretization strategy is a crucial aspect for resonant controllers

- The optimum discrete-time implementations have been assessed
Outline

1. Introduction
2. Effects of Discretization Methods on the Performance of Resonant Controllers
3. High Performance Digital Resonant Current Controllers Implemented with Two Integrators
   - Previous Schemes based on Two Integrators
   - Correction of Poles
   - Correction of Zeros
   - Experimental Results
   - Conclusions
5. Conclusions
Digital Resonant Current Controllers for Voltage Source Converters

High Performance Digital Resonant Current Controllers Implemented with Two Integrators

Previous Schemes based on Two Integrators

\[ G_{PRh}(s) \]

\[ G_{VPId}(s) \]

\[ G_{PRd}(s) \]

- Simple frequency adaptation
- Resonant poles deviation → steady-state error
- Leading angle \( \phi_h \) deviation →
  - anomalous peaks
  - instability
**Correction of Poles I**

- **Accurate** resonant poles: \(1 - 2z^{-1}\cos(h\omega_1T_s) + z^{-2}\)
- Schemes based on 2 int.: \(1 - 2z^{-1}(1 - \frac{h^2\omega_1^2T_s^2}{2}) + z^{-2}\)

**Proposed correction:**

\[
h^2\omega_1^2 \rightarrow C_h = 2 \sum_{n=1}^{n_T/2} \frac{(-1)^{n+1} h^{2n}\omega_1^{2n}T_s^{2n-2}}{(2n)!} = h^2\omega_1^2 - h^4\frac{\omega_1^4T_s^2}{12} + h^6\frac{\omega_1^6T_s^4}{360} + \ldots
\]

- **\(n_T = 4\)**: valid for most cases
- **\(n_T = 6\)**

---

**Graph:**
- Resonant frequency error (Hz) vs. \(h f_1/f_s\)
- Original, \(n_T = 2, 4, 6, 8\)
- \(n_T = 4\): valid for most cases
Correction of Poles I

- **Accurate** resonant poles: \(1 - 2z^{-1}\cos(h\omega_1 T_s) + z^{-2}\)
- Schemes based on 2 int.: \(1 - 2z^{-1}(1 - \frac{h^2 \omega_1^2 T_s^2}{2}) + z^{-2}\)

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\]

- \(n_T = 4\): valid for most cases
- \(n_T = 6\)

\[h_{f1}/f_s\] vs Resonant frequency error (Hz)

Original

- \(n_T = 2\)
- \(n_T = 4\)
- \(n_T = 6\)
- \(n_T = 8\)
Correction of Poles II

\[ G_{PR_h}(z) \]

\[ G_{VPI_h}(z) \]

\[ G_{dPR_h}(z) \]
**Correction of Zeros**

\[
G^d_{PR,h}(s) = K_{P_h} + K_{I_h} \cdot \frac{s \cos(\phi'_h) - h\omega_1 \sin(\phi'_h)}{s^2 + h^2\omega_1^2}
\]

- **\( \phi'_h \): target leading angle**
- **\( \phi_h \): actual leading angle**

**Ideally:**

\[
\phi_h = \phi'_h = -\angle G_{PL}(e^{jh\omega_1 T_s})
\]

**In previous literature:**

1. **Wrong target (2 samples)** → \( \phi'_h \neq -\angle G_{PL}(e^{jh\omega_1 T_s}) \)
2. **Discretization** → inaccuracy: \( \phi_h \neq \phi'_h \)
Correction of Zeros

\[ G_{PR,h}^d(s) = K_{P,h} + K_{I,h} \cdot \frac{s \cos(\phi'_h) - h\omega_1 \sin(\phi'_h)}{s^2 + h^2\omega_1^2} \]

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Correction of Zeros

\[ G_{PRh}^d(s) = KP_h + KI_h \cdot \frac{s \cos(\phi_h') - h\omega_1 \sin(\phi_h')}{s^2 + h^2\omega_1^2} \]

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Improvement in $\phi'_h$ Expression I

Objective:

$$\phi'_h \approx |\angle G_{PL}(e^{j\omega_1 T_s})|$$

2 samples: inaccurate

Proposed:

$$\phi'_h = \frac{\pi}{2} + \frac{3}{2} T_s$$

$\phi'_o$ $\lambda$
**Objective:**

\[ \phi'_h \approx \left| \angle G_{PL}(e^{j\omega_1 T_s}) \right| \]

- 2 samples: **inaccurate**

**Proposed:**

\[ \phi'_h = \frac{\pi}{2} + \frac{3}{2} T_s \]

\[ \phi'_o \]

\[ \lambda \]
**Objective:**

\[ \phi'_h \approx |\angle G_{PL}(e^{j\omega_1 T_s})| \]

- 2 samples: **inaccurate**

**Proposed:**

\[ \phi'_h = \frac{\pi}{2} + \frac{3}{2} T_s \]

\[ \phi'_o \approx \lambda T_s \]

- \[ f_{90} << f_s \]
- \[ h \approx \cdot T_s \]
- \[ o \approx h \]

\[ \angle G_{PL}(z) \]

- 2 samples \( \phi'_h \)
- Proposed \( \phi'_h \)
Improvement in $\phi'_h$ Expression II

Open-loop

Closed-loop

Less anomalous peaks

Greater phase margin

Proposed $\phi'_h$

2 samples $\phi'_h$

Open-loop

Closed-loop

Less anomalous peaks

Greater phase margin

Proposed $\phi'_h$

2 samples $\phi'_h$
Improvement in $\phi'_h$ Expression II

Open-loop

Closed-loop

Less anomalous peaks

Greater phase margin

Proposed $\phi'_h$

2 samples $\phi'_h$
Improvement in $\phi'_h$ Expression II

Open-loop

Closed-loop

Less anomalous peaks

Greater phase margin

Proposed $\phi'_h$

2 samples $\phi'_h$

Mag 80 60 40 20 0 -20 -40

Phase (deg) 180 90 0 -90 -180

Frequency (Hz) 1000 1200 1400 1600 1800 2000 2200
Improvement in $\phi'_h$ Expression II

Open-loop

Closed-loop

Less anomalous peaks

Greater phase margin

Proposed $\phi'_h$

2 samples $\phi'_h$

Phase (deg)

Frequency (Hz)

Magnitude (dB)

1000 1200 1400 1600 1800 2000 2200

-180 -90 0 90 180

-40 -20 0 20 40 60 80
Correction of $\phi_h$ Error Due to Discretization

1\textsuperscript{st} order Taylor approximation centered at 0

$$z^{-1} \left[ \cos (\phi'_h) - h\omega_1 T_s \sin (\phi'_h) \right] - z^{-2} \cos (\phi'_h) \quad \xrightarrow{\text{1st order Taylor approx. centered at 0}} \quad z^{-1} \cos (h\omega_1 T_s + \phi'_h) - z^{-2} \cos (\phi'_h)$$

Original delay compensation

Accurate

Optimized

$\Lambda \omega_1$ - $a_h$ - $d_h$: pre-calculated constants

Pag. 87-90

Alejandro Gómez Yepes

DTE, University of Vigo
Correction of $\phi_h$ Error Due to Discretization

1st order Taylor approximation centered at 0

$$z^{-1} \left[ \cos(\phi'_h) - h\omega_1 T_s \sin(\phi'_h) \right] - z^{-2} \cos(\phi'_h) = z^{-1} \cos(h\omega_1 T_s + \phi'_h) - z^{-2} \cos(\phi'_h)$$

Original delay compensation

Accurate

Optimized

1st order Taylor approximation centered at $h\omega_{1n}$

- $a_h - d_h$: pre-calculated constants

Pág. 87-90

Alejandro Gómez Yepes  
DTE, University of Vigo
Correction of $\phi_h$ Error Due to Discretization I

$1^{st}$ order Taylor approximation centered at 0

$$z^{-1} \left[ \cos (\phi'_h) - h\omega_1 T_s \sin (\phi'_h) \right] - z^{-2} \cos(\phi'_h) \quad \rightarrow \quad z^{-1} \cos (h\omega_1 T_s + \phi'_h) - z^{-2} \cos(\phi'_h)$$

Original delay compensation

Accurate

Optimized

$1^{st}$ order Taylor approximation centered at $h\omega_{1n}$

- $a_h - d_h$: pre-calculated constants
Correction of $\phi_h$ Error Due to Discretization II

1st order Taylor approx. centered at $0$

1st order Taylor approx. centered at $h \omega_{1n}$

Original

Accurate

Optimized
Correction of Poles I

$n_T = 2$
(no correction)
25.8 μs

$n_T = 6$
32.1 μs

$n_T = 4$
29.8 μs

$n_T = 8$
34.7 μs
Correction of Poles II

\[ n_T = 2 \]
(no correction)
25.8 \( \mu \)s

\[ n_T = 4 \]
29.8 \( \mu \)s

\[ n_T = 6 \]
32.1 \( \mu \)s

\[ n_T = 8 \]
34.7 \( \mu \)s

THD = 26.16%
Correction of Zeros

\[ f_1 = 25 \text{ Hz} \]

\[ f_1 = 90 \text{ Hz} \]
Conclusions of Chapter III

- Enhanced digital implementations of resonant controllers based on two integrators are contributed, with an optimized trade-off between accuracy and simplicity.
  - Correction of poles: improvement in steady-state error
  - Correction of zeros: improvement in stability margins
  - The advantages of the originals are maintained: low computational burden and easy frequency adaptation

- The steady-state error as a function of the order of Taylor series approximation of the poles has been studied. A fourth order is satisfactory for most cases

- An expression is proposed for the target leading angle, in order to achieve a better compensation of the plant phase lag
Digital Resonant Current Controllers for Voltage Source Converters
Analysis and Design of Resonant Current Controllers for VSCs by Nyquist Diagrams and Sensitivity Function

Outline

1. Introduction
2. Effects of Discretization Methods on the Performance of Resonant Controllers
3. High Performance Digital Resonant Current Controllers Implemented with Two Integrators
   - Limitations of Previous Approaches
   - Analysis of Stability Margins by Means of Nyquist Diagrams
   - Relation Between Closed-Loop Anomalous Peaks and Sensitivity Function
   - Minimization of Sensitivity Function
   - Experimental Results
   - Conclusions
5. Conclusions
Limitations of Previous Approaches

**PR controllers**

- \( PM_P \) is usually employed as indicator of stability. However, when there are more 0 dB crossings (e.g., high-power, selective control...), \( PM_P \) is no longer valid.
- Usually, phase margins are optimized. However:
  - the phase margin is a less reliable indicator than the sensitivity peak \( 1/\eta \)
  - closed-loop anomalous peaks are directly related to \( \eta \), not to phase margin

**VPI controllers**

- No methods have been proposed to measure and optimize proximity to instability and closed-loop anomalous peaks.
- Ambiguity in \( \phi'_n \) selection: leading angles of 1 & 2 samples have been proposed.
Nyquist Diagrams of PR Controllers I

\[ K_P T \, G_{PL}(z) \] (only proportional gain)

- \( K_P T \) defines \( \omega_c, PM_P, GM_P \) and \( \eta_P \)
- \( PM_h \leq PM_P, GM_h \leq GM_P \) and \( \eta_h \leq \eta_P \)
Nyquist Diagrams of PR Controllers I

\[ K_{PT} G_{PL}(z) \] (only proportional gain)

- \( K_{PT} \) defines \( \omega_c \), \( PM_P \), \( GM_P \) and \( \eta_P \)
- \( PM_h \leq PM_P \), \( GM_h \leq GM_P \) and \( \eta_h \leq \eta_P \)
Nyquist Diagrams of PR Controllers I

\[ \cdots, K_P T G_{PL}(z) \quad (\text{only proportional gain}) \]

\[ K_P T |G_{PL}(z)| \]

- \( K_P T \) defines \( \omega_c, PM_P, GM_P \) and \( \eta_P \)
- \( PM_h \leq PM_P, GM_h \leq GM_P \) and \( \eta_h \leq \eta_P \)
Nyquist Diagrams of PR Controllers I

- \( K_{PT} G_{PL}(z) \) (only proportional gain)
- \([K_{PT} + K_I R_1(z)] G_{PL}(z)\) (proportional+resonant term)

\[ K_{PT} |G_{PL}(z)| \quad \theta = \angle G_{PL}(z) \quad 0 \text{ rad/s} \]

\[ PM_P \quad \beta = \angle G_{PL}(z) \quad \omega_c \]

- \( K_{PT} \) defines \( \omega_c, PM_P, GM_P \) and \( \eta_P \)
- \( PM_h \leq PM_P, GM_h \leq GM_P \) and \( \eta_h \leq \eta_P \)
Nyquist Diagrams of PR Controllers

- $K_P G_{PL}(z)$ (only proportional gain)
- $[K_P + K_I R_1(z)] G_{PL}(z)$ (proportional+resonant term)

$K_P T$ defines $\omega_c$, $PM_P$, $GM_P$ and $\eta_P$

$PM_h \leq PM_P$, $GM_h \leq GM_P$ and $\eta_h \leq \eta_P$
Nyquist Diagrams of PR Controllers

- $K_P T G_{PL}(z)$ (only proportional gain)
- $[K_P T + K_I R_1(z)] G_{PL}(z)$ (proportional+resonant term)

- $K_P T$ defines $\omega_c, PM_P, GM_P$ and $\eta_P$
- $PM_h \leq PM_P, GM_h \leq GM_P$ and $\eta_h \leq \eta_P$
Nyquist Diagrams of PR Controllers II

- $GM_h > 0$ and $PM_P > 0$, but system is unstable
- $GM_h < 0$, but system is stable
- $\phi'_h = 0$ and $h \omega_1 > \omega_c$, but system is stable
Nyquist Diagrams of PR Controllers II

- $GM_h > 0$ and $PM_P > 0$, but system is unstable
- $GM_h < 0$, but system is stable
- $\phi'_h = 0$ and $h\omega_1 > \omega_c$, but system is stable
Nyquist Diagrams of PR Controllers II

- $GM_h > 0$ and $PM_P > 0$, but system is unstable
- $GM_h < 0$, but system is stable
- $\phi'_h = 0$ and $h\omega_1 > \omega_c$, but system is stable
Nyquist Diagrams of PR Controllers II

- $GM_h > 0$ and $PM_P > 0$, but system is unstable
- $GM_h < 0$, but system is stable
- $\phi'_h = 0$ and $h\omega_1 > \omega_c$, but system is stable
Nyquist Diagrams of PR Controllers III

ηₙ provides more information about the actual proximity to instability than PMₙ.
Nyquist Diagrams of PR Controllers IV

- $\phi'_h = -\angle G_{PL}(e^{j\omega_1 T_s})$ is usually pursued, but it is not the optimum.
- **Objective**: $\phi'_h$ such that $\eta_h$ becomes maximum, i.e., $\eta_h = \eta_P \ \forall h$.

![Nyquist Diagram](image-url)
Nyquist Diagrams of PR Controllers V

\( \eta \) defines:

- Proximity to instability
- Oscillations during transients (damping)
- Maximum steady-state error

\[
\max\{|S(z)|\} = \frac{1}{\eta} \leq \begin{cases} 
|D(z)| \leq \eta \\
S(z) = \frac{E(z)}{I^*(z)} = \frac{1}{D(z)}
\end{cases}
\]

\( \eta_P \) is set by \( K_{PT} \):

\[
K_{PT} = F_1 (\eta_P, T_s, R_F, L_F)
\]
**Nyuquist Diagrams of VPI Controllers**

- $|G_{PL}(z)|$ is cancelled
- $\phi_h = \phi_h' + \arctan(h\omega_1 L_F / R_F)$

Greater stability margins and easier design than PR

Stable $\iff \gamma_h > 0 \ \forall h$
Relation in PR Controllers

\[ S(z) = \frac{E(z)}{I^*(z)} \]

\[ C_L(z) = \frac{I(z)}{I^*(z)} \]

- A: \( h\omega_1 \)
- B: \( D(e^{j\omega B T_s}) = \eta_h \)
Relation in PR Controllers

\[ S(z) = \frac{E(z)}{I^*(z)} \]

\[ C_L(z) = \frac{I(z)}{I^*(z)} \sim \frac{1}{\eta_h} \]

- **A**: \( h_\omega_1 \)
- **B**: \( D(e^{j\omega_BT_s}) = \eta_h \)
Relation in VPI Controllers

\[ S(z) = \frac{E(z)}{I^*(z)} \]

\[ |S(e^{j\omega_B T_s})| = \frac{1}{\eta_h} \]

\[ C_L(z) = \frac{I(z)}{I^*(z)} \]

\[ |C_L(e^{j\omega_B T_s})| \approx \frac{1}{\eta_h} \]

\[ A: \ h\omega_1 \]

\[ B: \ D(e^{j\omega_B T_s}) = \eta_h \]
Minimization of $S(z)$ in PR Controllers

**Objective:** $\phi'_h$ such that $\eta_h$ becomes maximum, i.e., $\eta_h = \eta_P \forall h$

**Solution:** $\phi'_h = -\angle G_{PL}(e^{jh\omega_1 T_s}) + \angle D(e^{jh\omega_1 T_s})$
Minimization of $S(z)$ in VPI Controllers

Optimum: $\phi'_h = \frac{3}{2} T_s \Leftrightarrow \begin{cases} \max. \eta_h \Rightarrow \gamma_h = \frac{\pi}{2} \Rightarrow \phi_h = -\angle G_{PL}(e^{j\omega_1 T_s}) \\ \phi_h = \phi'_h + \arctan(h\omega_1 L_F / R_F) \end{cases}$

![Diagram showing Nyquist plot with optimal phase margin and gain margin]
Test I

**Objective of Test I**: prove the **sensitivity minimization** around $\omega_1$ achieved by the **proposed** $\phi'_h$ expressions.
Test 1 (PR Controller)

$\phi_n' = 0$

Proposed $\phi_n'$
Test I (VPI Controller)

$\phi'_h = 0$

Proposed $\phi'_h$
Test II (Transient)

Experimental Results
Test II (Transient)

PR

VPI
Test II (Steady-State)

THD = 28.4%

PR

THD = 3.4%

VPI

THD = 3.5%
Conclusions of Chapter IV

- PR and VPI controllers, including delay compensation, are analyzed by means of Nyquist diagrams. The effect of each freedom degree on the trajectories is studied, and their relation with the sensitivity function and its peak value is assessed.

- Optimization of the sensitivity peak permits to achieve a better performance and stability in resonant controllers than optimization of the gain or phase margins.

- A systematic method, supported by closed-form analytical expressions, is proposed to optimize:
  - stability
  - avoidance of closed-loop anomalous peaks
  - transient response (greater damping of frequencies at which the trajectory is closer to the critical point)
Digital Resonant Current Controllers for Voltage Source Converters

Conclusions

Outline

1. Introduction
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5. Conclusions
   - Conclusions
   - Future Research
Conclusions

- An exhaustive study and comparison of discretization techniques applied to resonant controllers has been presented, in terms of
  - steady-state error
  - stability
- Implementations based on two integrators, that overcome the issues of the original ones, have been proposed. They achieve an optimized tradeoff between
  - accuracy
  - simplicity
- It is proved that to minimize the sensitivity peak permits a better performance and stability in resonant controllers than to maximize the gain or phase margins. A systematic method is proposed to obtain
  - high stability
  - reduced closed-loop anomalous peaks
  - reduced oscillations in transients
Future Research

- Optimization of **transient** response for **distributed** power generation systems

- **Torque ripple** minimization

- Injection of current harmonics in **multi-phase** drives for **fault-tolerance** operation and **increase of average torque**

- Active compensation of undesired current components caused by **dead-time** in **multi-phase** converters
Digital Resonant Current Controllers for Voltage Source Converters

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“Doctor Europeus” mention